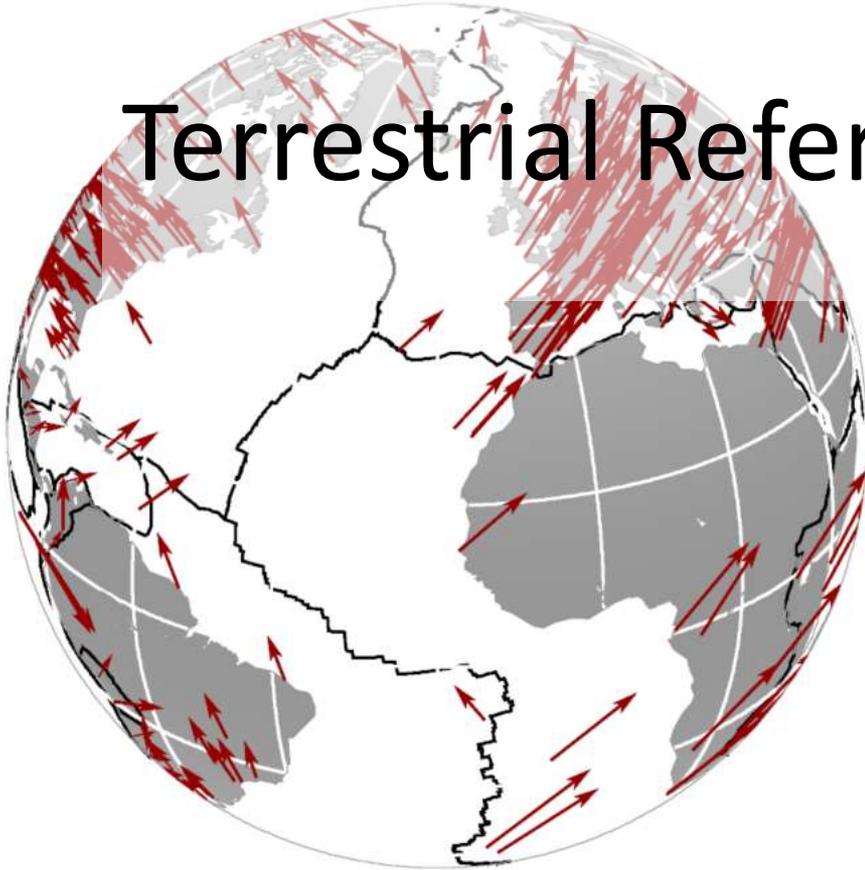


Terrestrial Reference Frames



EGU campfire - Geodesy 101

December 1st 2022



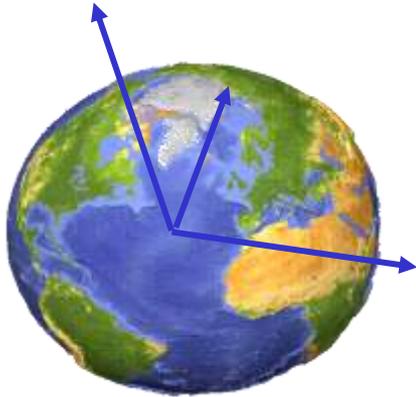
Xavier Collilieux

xavier.collilieux@ensg.eu

Acknowledgment:

Z. Altamimi, J. Legrand, P. Rebischung,

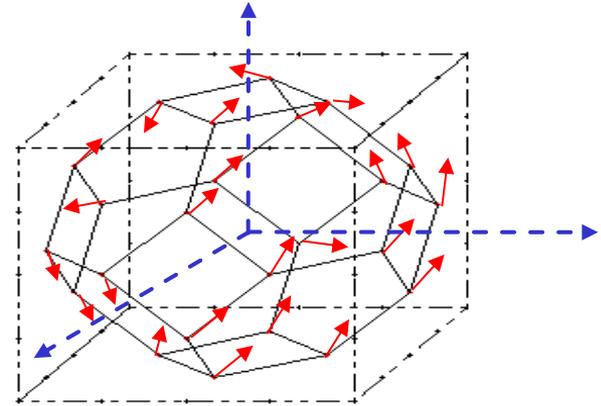
R. Rietbroek, L. Métivier



Concept:

(=Terrestrial Reference System)

- Mathematical concept



Real world: **Terrestrial Reference Frame (TRF)**

- Table of coordinates of materialized points (=reference points).
May include velocities (=dynamic TRF) + other terms + statistical information
- Axes can be accessed by positioning objects with respect to reference points

Why so many terrestrial reference frames?

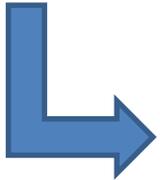
Example of Terrestrial Reference Frames:

NAD 83, OSGB 1970, RGF93

ED 50, ETRF2014, ETRF2000, ...

WGS84(2139), GTRF16v01, GTRF, PZ-90.11, CGCS2000

ITRF2020, ITRF2014, ITRF2008, ITRF2005, ...



- Network coverage, new permanent instruments
- Earth's non predictable deformations (Earthquake)
- Measurement technique improvement (infrastructure, processing), computation strategy
- Limited Coordinate precision (see later)

How to determine coordinates?

Space geodetic techniques. VLBI : Very Long Baseline Interferometry

Principle:

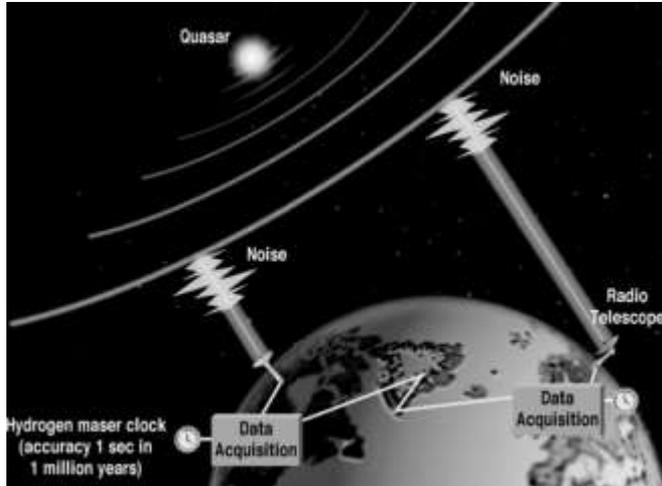


Image source: IVS web site, <https://ivscc.gsfc.nasa.gov/about/vlbi/whatis.html>



Contribution

- Link to the celestial frame: Earth rotation
- Long history

Structure:

Activity coordinated by the International VLBI Service (IVS):

- Scheduling
- Station operations
- Data centers
- Correlation Centers
- Analysis Centers etc...

Nothnagel A., T. Artz, D. Behrend, Z. Malkin (2017), "International VLBI Service for Geodesy and Astrometry – Delivering high-quality products and embarking on observations of the next generation", Journal of Geodesy, 91(7):711–721, DOI:10.1007/s00190-016-0950-5

How to determine coordinates?

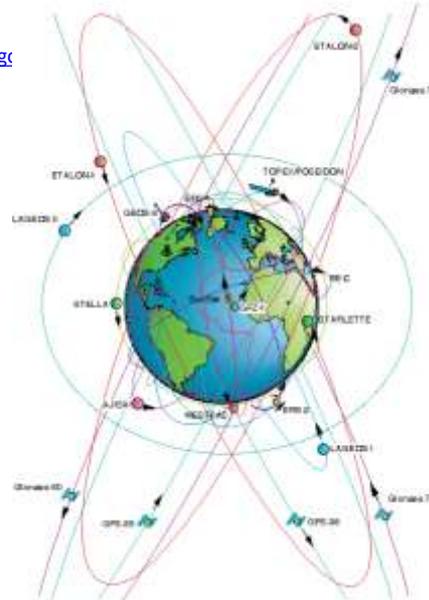
Space geodetic techniques. Satellite Laser Ranging (SLR)



Herstmonceux SLR station

Source: <http://sgf.rgo.ac.uk/work.html>

Lageos satellite
Source: <https://lageos.gsfc.nasa.gov>



Satellite mission with SLR reflectors.

Source:

https://ilrs.gsfc.nasa.gov/missions/satellite_missions/index.html



Contribution

- Origin of the Frame (CM)
- Long history
- POD/orbit evaluation

Structure:

Activity coordinated by the International SLR Service (ILRS)

Pearlman, M.R., Noll, C.E., Pavlis, E.C. et al. (2019) The ILRS: approaching 20 years and planning for the future. J Geod 93:2161–2180. DOI: <https://doi.org/10.1007/s00190-019-01241-1>

How to determine coordinates?

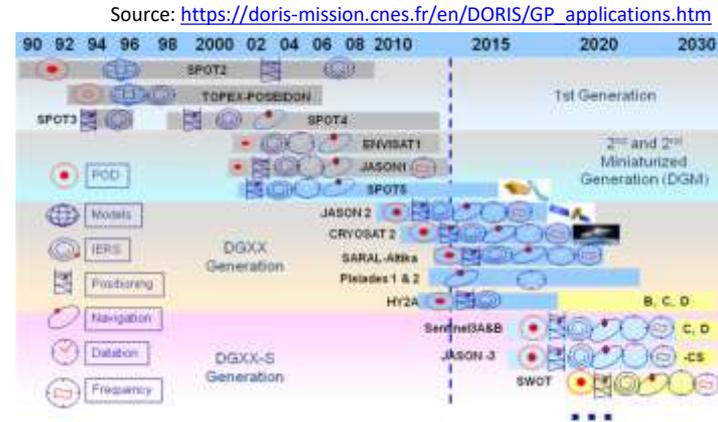
Space geodetic techniques. DORIS = Doppler & Radiopositioning Integrated by Satellite

Contribution

- Spatial coverage over its full history
- POD, ITRF dissemination



Source: <https://space-geodesy.nasa.gov/techniques/DORIS.html>



Structure:

Activity coordinated by the International DORIS Service (IDS)

Willis, P.; Lemoine, F.G.; Moreaux, G. et al. (2016), The International DORIS Service (IDS), recent developments in preparation for ITRF2013, IAG SYMPOSIA SERIES, 143, 631-639, DOI: 10.1007/1345_2015_164



Source: <https://ids-doris.org/network/site/logs/station.html?code=KERGUELEN>



How to determine coordinates?

Space geodetic techniques. GNSS

GPS, Glonass, Beidou, Galileo,
QZSS, IRNSS



Credit: ESA-P. Carril
https://www.esa.int/ESA_Multimedia/Images/2014/07/Galileo_constellation



Noto GNSS station. Source:
<https://epncb.oma.be/networkdata/site/info4onestation.php?station=NOT100ITA>



Contribution

- Temporal resolution
- Network
- ITRF dissemination

Structure:

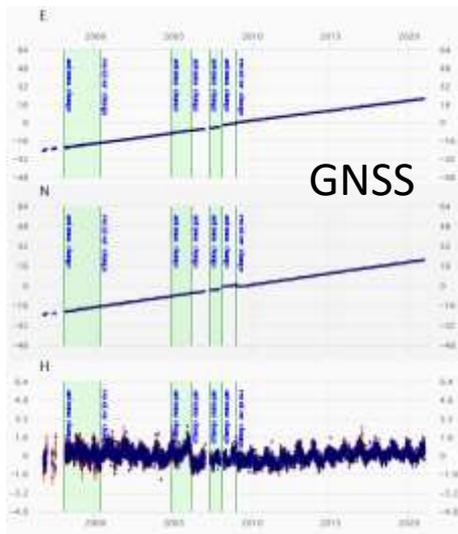
Activity coordinated by the International GNSS Service (IGS)

Johnston, G., Riddell, A., Hausler, G. (2017). The International GNSS Service. Teunissen, Peter J.G., & Montenbruck, O. (Eds.), Springer Handbook of Global Navigation Satellite Systems (1st ed., pp. 967-982). Cham, Switzerland: Springer International Publishing. DOI: 10.1007/978-3-319-42928-1.

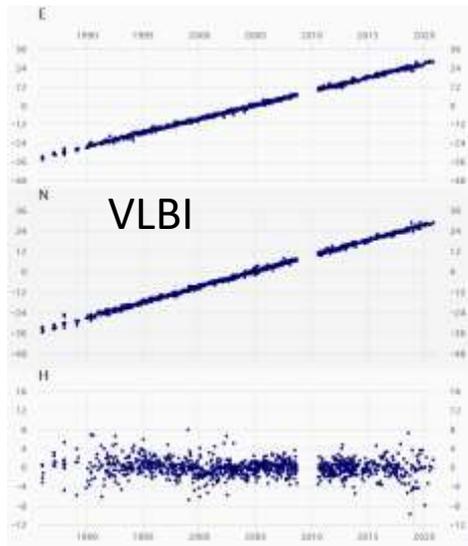
How to determine coordinates?

Example of position time series

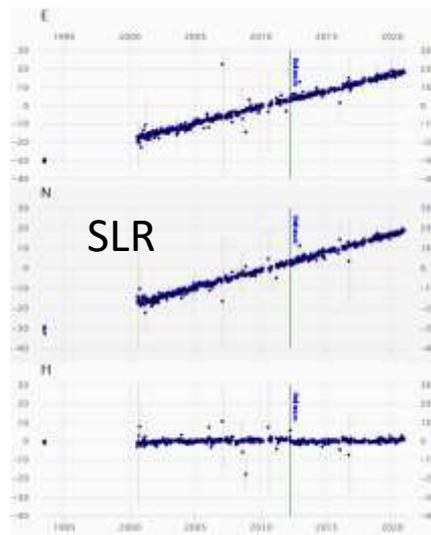
Image source: Muller J-M, J-C. Poyard (2014),
Hartebeesthoek local tie survey, RT/G 199



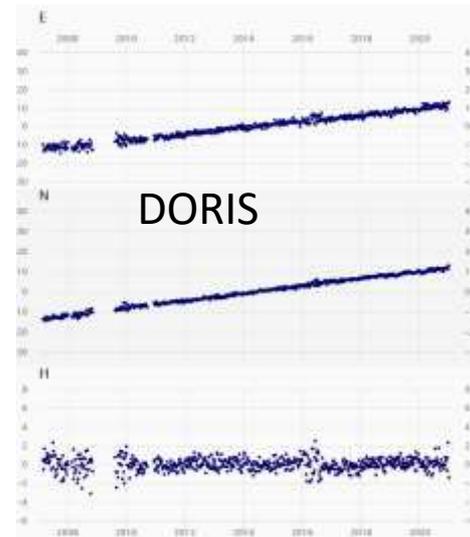
GNSS



VLBI



SLR



DORIS

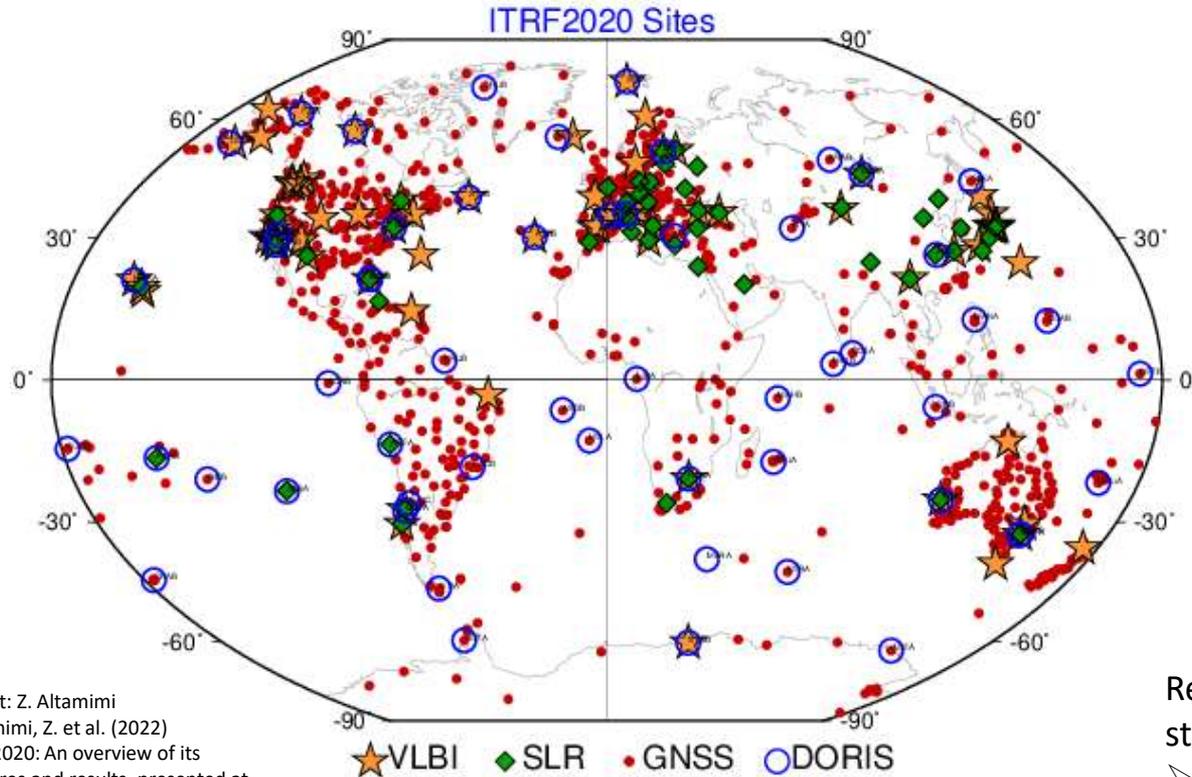
Plots available at: <https://itrf.ign.fr/en/timeseries>

Data source: IAG technique services and ITRF2020 analysis (Altamimi et al., 2022)

- Hellmers H., Modiri S., Thaller D., Gipson J., Blossfeld M., Seitz M., Bachmann, S. (2022) The IVS contribution to ITRF2020, technical report, itrf.ign.fr
- Moreaux G., P. Štěpánek, H. Capdeville, F. Lemoine, M. Otten (2022), The DORIS contribution to ITRF2020, technical report, itrf.ign.fr
- Pavlis E., Luceri V. (2022), The ILRS contribution to ITRF2020, technical report, itrf.ign.fr
- Rebischung P. (2022) IGS contribution to ITRF2020, technical report, itrf.ign.fr

How to determine coordinates?

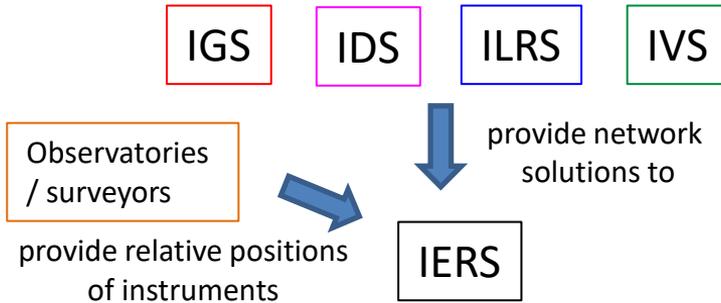
Whole network processed by the International Association of Geodesy (IAG) technique services for ITRF2020



Credit: Z. Altamimi
Altamimi, Z. et al. (2022)
ITRF2020: An overview of its
features and results, presented at
REFAG 2022, oct 2022



Relative positions determined by
standard surveying at co-location sites.
➤ 253 such vectors in ITRF2020



The **International Earth Rotation and Reference Systems Service (IERS)** :
 in charge of processing the **International Terrestrial Reference Frame (ITRF)** which gives a physical access to the **International Terrestrial Reference System (ITRS)**.

IERS functions as one of the International Association of Geodesy (IAG) Scientific Services.

<https://www.iers.org>

International Earth Rotation and Reference Systems Service

Organization | Data / Products / Tools | Publications | Science background | News / Meetings

Earth orientation data

ICRF

ICRS

ITRF

ITRS

Geophysical fluids data

Conventions

Data analysis tools

Search website: search item >>

Search IERS products: Product search

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- Privacy statement
- Contact

IERS > Data / Products / Tools

Data / Products / Tools

The IERS provides data on Earth orientation, on the International Celestial Reference System/Frame, on the International Terrestrial Reference System/Frame, and on geophysical fluids. It maintains also Conventions containing models, constants and standards.

For regular information on changes concerning IERS data, products and tools please [register as IERS user](#) and subscribe to [IERS Messages](#).

Earth orientation data

IERS provides rapid, monthly and long term earth orientation data as well as leap second announcements and announcements of DUT1 in the form of bulletins or data files. [more...](#)

ICRF / ICRS

The International Celestial Reference Frame (ICRF) consists of equatorial coordinates of extragalactic radio sources observed with VLBI, realizing an ideal reference system, the International Celestial Reference System (ICRS). [ICRF](#) [ICRS](#)

ITRF / ITRS

The International Terrestrial Reference Frame (ITRF) is a set of points with their 3-dimensional cartesian coordinates which realize an ideal reference system, the International Terrestrial Reference System (ITRS). [ITRF](#) [ITRS](#)

Geophysical fluids data

Global geophysical fluids data provide information related to Earth rotation variation, gravity field variation and geocenter motion that are caused by mass transports in the global geophysical fluids (atmosphere, oceans, hydrology, tides, mantle, core). [more...](#)

IERS Conventions

The IERS Conventions define the standard reference system to be used by the IERS. They contain conventional models, constants and standards. [more...](#)

Frame definition. What needs?

➤ Various needs:

○ Mapping, civil engineering

○ Navigation

○ National reference frame

○ Earth sciences: Earthquakes, volcanology, deformation monitoring

○ Astronomy : Observatory position, Earth's rotation in space

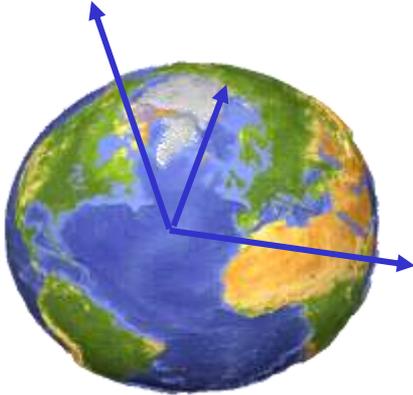
○ Precise Orbit Determination (POD): altimetry satellite, GNSS satellites

Etc...

➤ Requirements (GGOS = Global Geodetic Observing System):
1 mm and 0.1 mm/yr in TRF frame parameters

Plag, H. P., & Pearlman, M. (2007). The **Global Geodetic Observing System**: Meeting the Requirements of a Global Society on a Changing Planet in 2020 The Reference Document. *Int. Assoc. Geod.*

Frame definition. ITRS



- Origin :
Earth center of mass (CM) (solid Earth + fluid layers)
- Scale:
 - SI units
 - Consistent with TCG (Geocentric Coordinate Time) through relativistic modeling
- Orientation
 - At reference epoch : conventional
 - Time evolution: **no-net-rotation** of horizontal tectonic motions over the whole Earth which could be written as

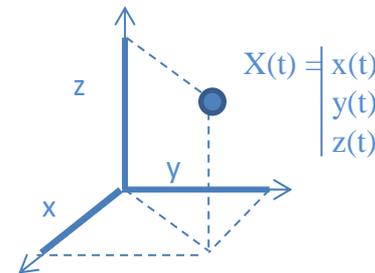
$$\vec{h}_C = \iint_C \vec{r} \wedge \left[\frac{d}{dt} \vec{r} \right]_{ITRS} dM = \vec{0}$$

↙
Earth's crust

Boucher, C., Current Intercomparisons Between CTS's, in Reference Frames in Astronomy and Geophysics, edited by J. Kovalevsky, I. I. Mueller, and B. Kolaczek, Kluwer Academic Publisher, Dordrecht, pp.327 – 343, 1989.

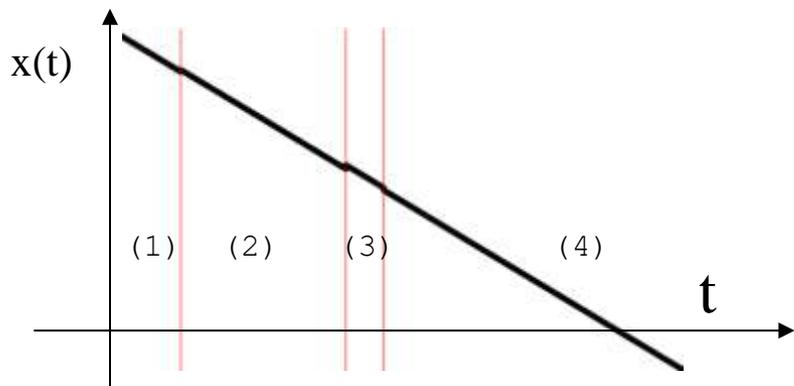
ITRF coordinates

ITRF = table of coordinates



Example: coordinate of one point

Reference epoch	t (year)	x (m)	y (m)	z (m)	vx (m/y)	vy (m/y)	vz (m/y)	tmin	tmax	id
<u>Ex: the reference epoch of ITRF2014 is 2010.0</u>	2010.0	4202777.3053	171368.0862	4778660.2517	-.01305	0.01758	0.01031			(1)
	2010.0	4202777.3058	171368.0882	4778660.2528	-.01305	0.01758	0.01031	96:277	03:113	(2)
	2010.0	4202777.3056	171368.0882	4778660.2528	-.01312	0.01760	0.01074	03:113	04:295	(3)
	Etc									

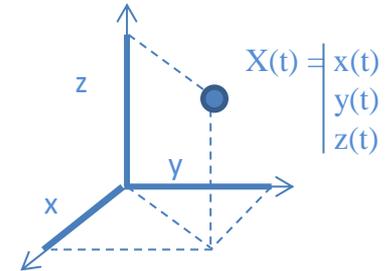


Example: What is the x coordinate at epoch 2002.5?

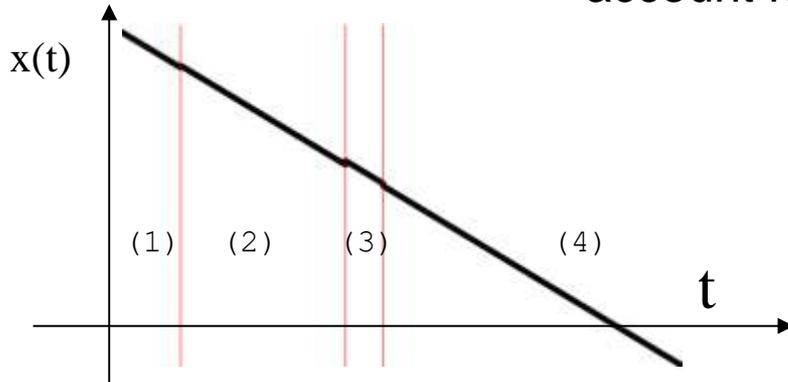
$$\begin{aligned}
 x(2002.5) &= x(2010.0) + (2002.5 - 2010.0) \cdot vx(2002.5) \\
 &= 4202777.3058 - 7.5 \cdot (-.01305) \\
 &= 4202777.4037
 \end{aligned}$$

ITRF coordinates

ITRF = table of coordinates



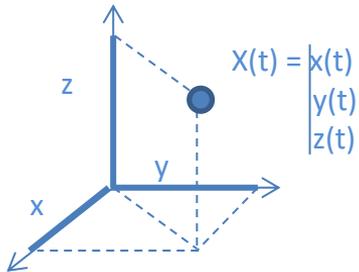
What kind of deformation does this kinematic model account for?



- Tectonic motion in stable part of the tectonic plates
 - Post-Glacial Rebound
 - Earthquake Co-seismic motion
 - Post-seismic motion in Earthquake (far-field areas)
- + Instrument related discontinuities !

This description is valid up to ITRF2008: ITRF94, ..., ITRF96, ... ITRF2008

Relationship between frame coordinates and « instantaneous coordinates »

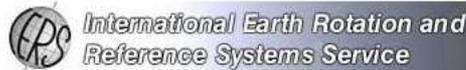


$$X(t) = X_{\text{ITRF}}(t) + \Delta X(t)$$

Well modeled crust deformation:

- Solid Earth tides
- Pole tides
- Tidal Ocean tide loading
- Atmospheric tide loading

<https://www.iers.org/IERS/EN/DataProducts/Conventions/conventions.html> LOGIN



Organization

Data / Products / Tools

Publications

IERS Conventions (2010). Gérard Petit and Brian Luzum (eds.). (IERS Technical Note ; 36) Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie, 2010. 179 pp., ISBN 3-89888-989-6

IERS
Technical
Note

No. 36



Some models include a conventional part.
Ex: solid Earth tides

7 Displacement of reference points

Models describing the displacements of reference points due to various effects are provided. In the following, three kinds of displacements are distinguished:

- Conventional displacements of reference markers on the crust (see Section 7.1) relate the regularized positions $X_R(t)$ of the reference points (see Chapter 4) to their conventional instantaneous positions. Generally these conventional instantaneous positions are used in data analyses as *a priori* coordinates for subsequent adjustment of observational data. They include tidal motions (mostly ocean tide loading, solid Earth tides) and other conventional modeled effects.

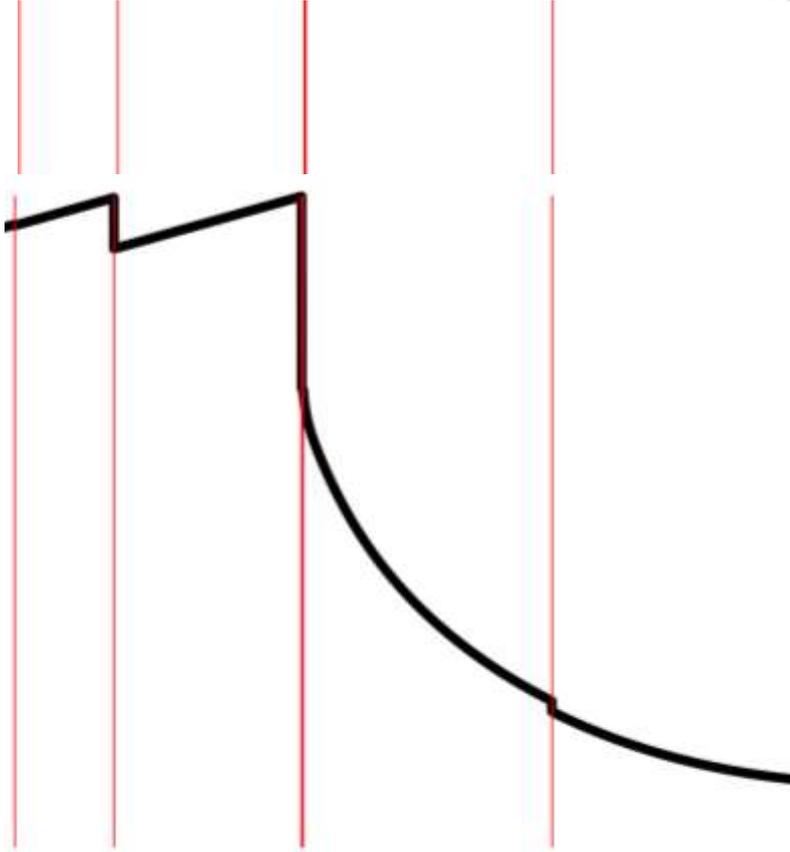
Conventions

IERS Conventions

IERS Conventions

ITRF coordinates

Kinematic model in ITRF2014



$$\begin{aligned} X_{ITRF}(t) = & X(t_0) + V(t-t_0) \\ & + \sum_i dX_i \cdot H(t-t_i) + \sum_j dV_j \cdot H(t-t_j) \\ & + d_{PSD}(t) \end{aligned}$$

Position discontinuities at t_i

Velocity changes at t_j

Post-seismic deformation model at t_i

ITRF coordinates

Post-seismic displacements

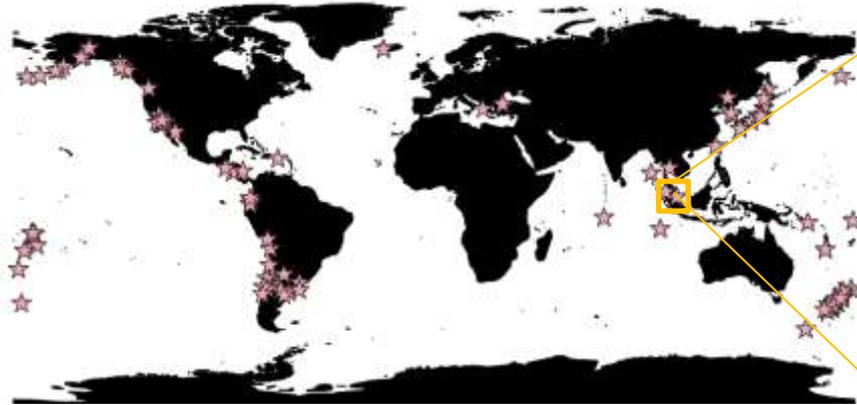
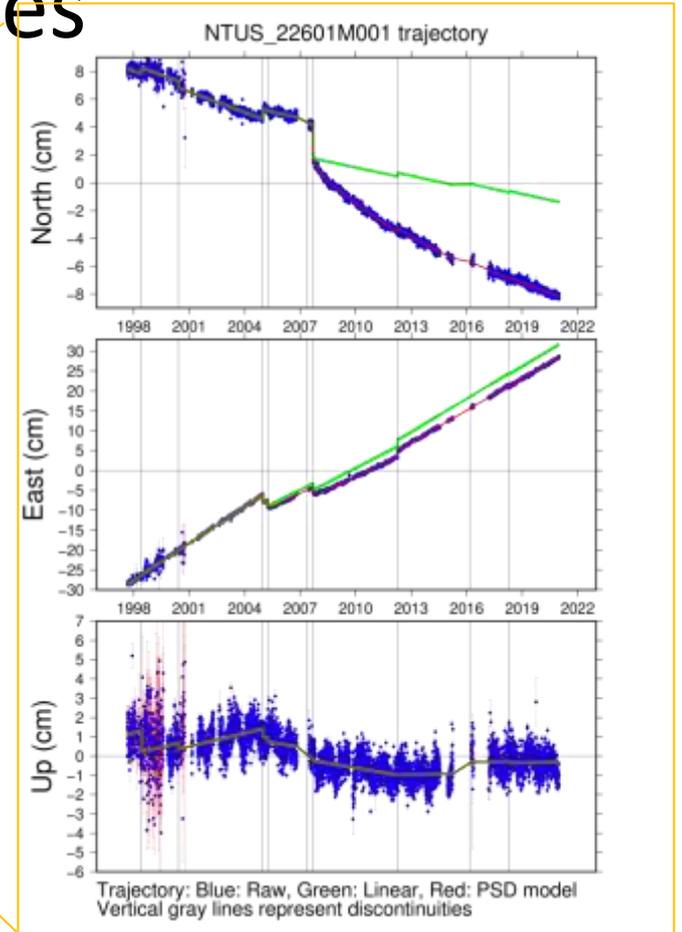


Fig. Stations with post-seismic motion modelling in ITRF2020



Post-seismic displacements

How to compute the post-seismic displacement?

$$d_{\text{PSD}}(t) = ?$$

- Models provided for East, North and Height separately
- Composed of up to two functions

$$\sum_{i=1}^{n^l} A_i^l \log\left(1 + \frac{t - t_i^l}{\tau_i^l}\right) + \sum_{i=1}^{n^e} A_i^e \left(1 - e^{-\frac{t - t_i^e}{\tau_i^e}}\right)$$

- Coefficients A_i^l A_i^e t_i^l t_i^e τ_i^l τ_i^e are provided
- Displacements need to be converted in Cartesian x,y,z using a rotation matrix (which depends on point longitude λ and latitude ϕ)

$$R(\lambda, \phi) = \begin{pmatrix} -\sin(\lambda) & \cos(\lambda) & 0 \\ -\sin(\phi) \cos \lambda & -\sin(\phi) \sin \lambda & \cos \phi \\ \cos \phi \cos \lambda & \cos \phi \sin \lambda & \sin \phi \end{pmatrix}$$

More info at <http://itrf.ign.fr>

ITRF2020P: Equations of post-seismic deformation models

After an Earthquake, the position of a station during the post-seismic trajectory, $X_{\text{PSD}}(t)$, at an epoch t could be written as:

$$X_{\text{PSD}}(t) = X(t_0) + \dot{X}(t - t_0) + \Delta X_{\text{PSD}}(t) \quad (1)$$

where \dot{X} is the station linear velocity vector, and $\Delta X_{\text{PSD}}(t)$ is the total sum of the post-seismic deformation (PSD) corrections at epoch t . For each component $L \in \{E, N, U\}$, we note δL the total sum of PSD corrections expressed in the local frame at epoch t :

$$\delta L(t) = \sum_{i=1}^{n^l} A_i^l \log\left(1 + \frac{t - t_i^l}{\tau_i^l}\right) + \sum_{i=1}^{n^e} A_i^e \left(1 - e^{-\frac{t - t_i^e}{\tau_i^e}}\right) \quad (2)$$

where:

n^l : Number of logarithmic terms of the parametric model

n^e : Number of exponential terms of the parametric model

A_i^l : Amplitude of the i^{th} logarithmic term

A_i^e : Amplitude of the i^{th} exponential term

τ_i^l : Relaxation time of the i^{th} logarithmic term

τ_i^e : Relaxation time of the i^{th} exponential term

t_i^l : Earthquake time(date) corresponding to i^{th} logarithmic term

t_i^e : Earthquake time(date) corresponding to i^{th} exponential term

The variance of $\delta L(t)$ is given by:

$$\text{var}(\delta L) = C \cdot \text{var}(\theta) \cdot C^T \quad (3)$$

where θ is the vector of parameters of the post-seismic deformation model:

$$\theta = [A_1^l, \tau_1^l, \dots, A_{n^l}^l, \tau_{n^l}^l, A_1^e, \tau_1^e, \dots, A_{n^e}^e, \tau_{n^e}^e]$$

The elements of the matrix C are computed by the following formulas:

$$\frac{\partial \delta L}{\partial A_i^l} = \log\left(1 + \frac{t - t_i^l}{\tau_i^l}\right) \quad (4)$$

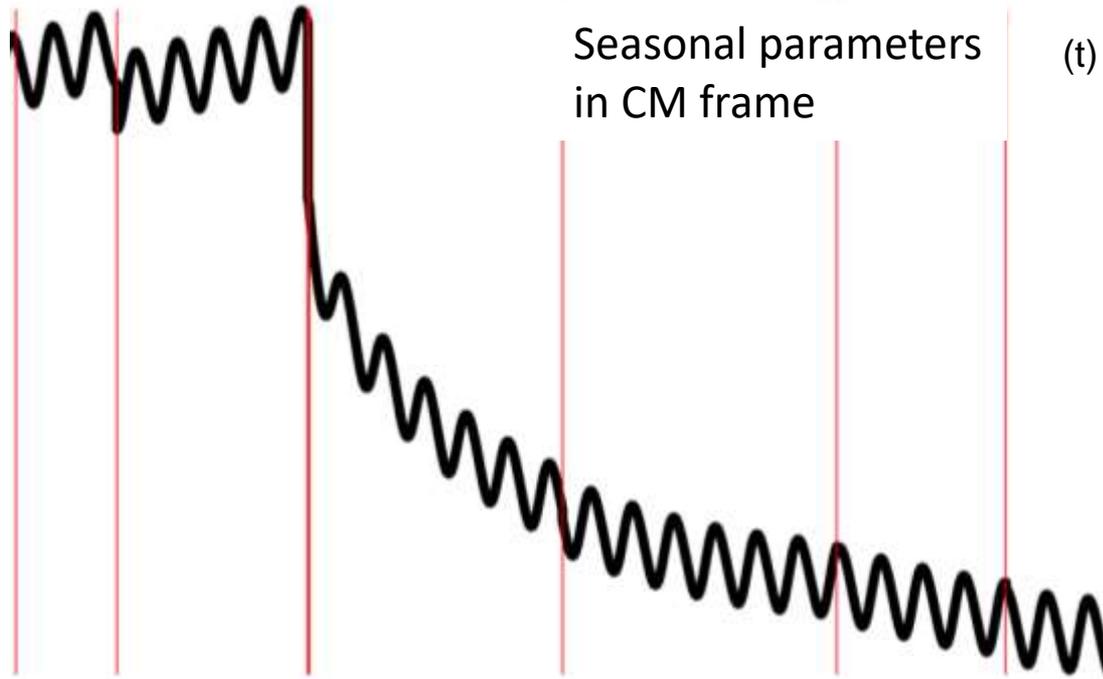
$$\frac{\partial \delta L}{\partial \tau_i^l} = \frac{A_i^l (t - t_i^l)}{(\tau_i^l)^2 \left(1 + \frac{t - t_i^l}{\tau_i^l}\right)} \quad (5)$$

$$\frac{\partial \delta L}{\partial A_i^e} = 1 - e^{-\frac{t - t_i^e}{\tau_i^e}} \quad (6)$$

$$\frac{\partial \delta L}{\partial \tau_i^e} = \frac{A_i^e (t - t_i^e) e^{-\frac{t - t_i^e}{\tau_i^e}}}{(\tau_i^e)^2} \quad (7)$$

ITRF coordinates

Kinematic model in **ITRF2020**



Seasonal parameters
in CM frame

$$\begin{aligned}
 \mathbf{x}(t) = & \mathbf{X}(t_0) + \mathbf{V}(t-t_0) \\
 & + \sum_i d\mathbf{X}_i \cdot H(t-t_i) + \sum_j d\mathbf{V}_j \cdot H(t-t_j) \\
 & + d_{\text{PSD}}(t) \quad \leftarrow \text{Post-seismic deformation model at } t_i \\
 & + d_{\text{seasonal CM}}(t)
 \end{aligned}$$

Position discontinuities at t_i

Velocity discontinuities at t_j

Periodic displacements

$$d_{\text{seasonal CM}}(t) = a_i \cos(2\pi t) + b_i \sin(2\pi t) \leftarrow \text{annual} \\ + c_i \cos(4\pi t) + d_i \sin(4\pi t) \leftarrow \text{Semi-annual}$$

with t in decimal year

$$d_{\text{seasonal CM}}(t) = d_{\text{seasonal CF}}(t) - \underbrace{T_{\text{seasonal}}(t)}_{\text{geocenter motion : CM w.r.t CF}}$$

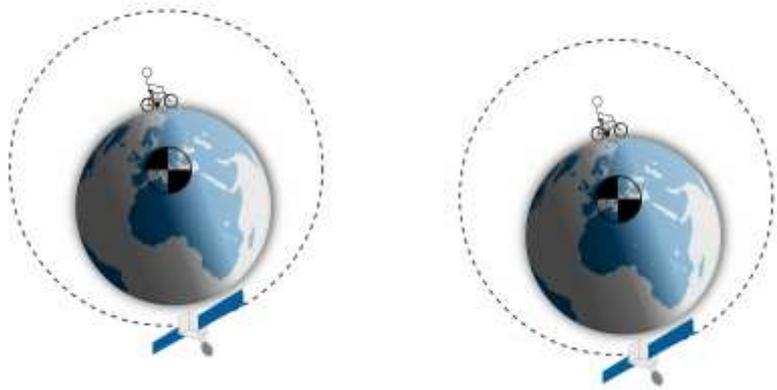


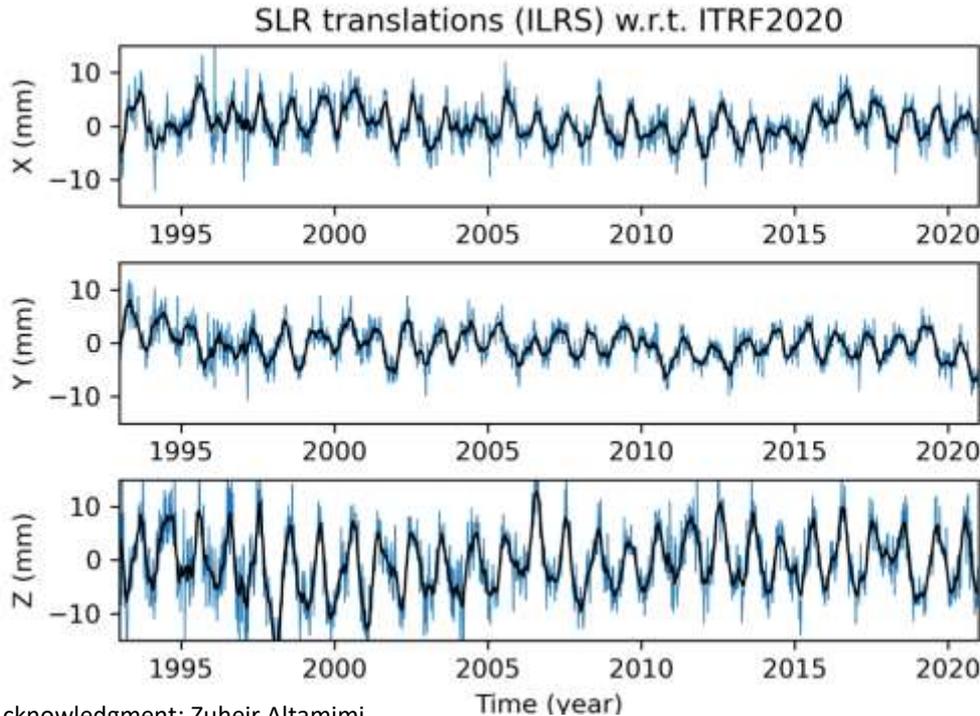
Figure by Roelof Rietbroek. See the post: <https://wobbly.earth/post/geocenter/>

By definition, in the center of figure frame (CF), the averaged displacement over the whole Earth's surface is zero.

Blewitt, G. (2003). Self-consistency in reference frames, geocenter definition, and surface loading of the solid Earth. *Journal of geophysical research: solid earth*, 108(B2).

ITRF2020

Magnitude of the geocenter motion: Net-translation of SLR network over time



Acknowledgment: Zuheir Altamimi

A specific seasonal geocenter motion model has been computed from ITRF2020 analysis.

→ This model is in good agreement with recent geocenter motion estimates in particular Wu et al. (2020).

Rebischung, P., Z. Altamimi, X. Collilieux, L. Métivier and K. Chanard (2022) ITRF2020 seasonal geocenter motion model, REFAG 2022, oct 2022

Wu, X., Haines, B. J., Heflin, M. B., & Landerer, F. W. (2020). Improved global nonlinear surface mass variation estimates from geodetic displacements and reconciliation with GRACE data. *Journal of Geophysical Research: Solid Earth*, 125(2), e2019JB018355.

ITRF2020

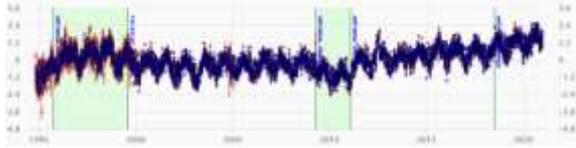


Fig: height displacement at Potsdam observed by GNSS
IGS data submitted for ITRF2020 (source: itrfr.ign.fr)

Why and when using seasonal displacements?

CAUTION! ITRF2020 seasonal parameters contains seasonal non-tidal loading deformations

- To determine satellite orbits (Precise Orbit Determination). Need geocenter motion so displacement in CM frame
- Alignment of space geodetic solutions to preserve seasonal signals in the aligned solution

So which frame for seasonal?

- if CM: the aligned solution will contain a net translational motion (=ITRF2020 geocenter motion model)
- if CF : no-net translational motion

ITRF2020

<https://itrf.ign.fr/en/solutions/itrf2020>

The screenshot shows the ITRF2020 website interface. On the left is a navigation menu with options like Home, Description, Input data, and Station list. The main content area is titled 'ITRF2020' and includes a 'Description' section explaining the system and an 'Input data' section with a table of station parameters.

ITRF2020 STATION POSITIONS AT EPOCH 2000.0 AND VELOCITIES

IGRS STATIONS

BONES NR.	SITE NAME	TECH. ID.	K/A/N	V/W	Z/H	Sigma	SOLN	DATA_START
188825001	Graess (OCA)	SLR 7833	451091.526	556159.691	4380358.534	0.001 0.001 0.001		
188825002			-0.0142	0.0180	0.0156	-0.001 -0.001 -0.001		
188825003	Graess (OCA)	SLR 7835	451092.009	556156.178	4383355.178	0.001 0.001 0.001	1	0000000000
188825004			-0.0142	0.0180	0.0156	-0.001 -0.001 -0.001		
188825005	Graess (OCA)	SLR 7844	451092.008	556156.178	4383355.178	0.001 0.001 0.001	2	0110000000
188825006			-0.0142	0.0180	0.0156	-0.001 -0.001 -0.001		
188770002	SIACCOI	SLR 7848	4680991.918	7248001.667	4238673.644	0.003 0.004 0.003		
188770003			-0.0142	0.0180	0.0099	-0.013 0.016 0.010		
189835001	HITSANAKVI	SLR 7885	2892595.256	1311007.525	5512028.377	0.009 0.009 0.007		
189835002			-0.0157	0.0164	0.0246	-0.008 -0.007 -0.006		
189835003	HITSANAKVI	SLR 7886	2892607.099	1311015.192	5512198.733	0.003 0.003 0.002		
189835004			-0.0157	0.0164	0.0246	-0.008 -0.008 -0.007		
118805002	GRAZ LUSTRUEHEL	SLR 7838	4194426.379	1162094.178	4647246.733	0.001 0.001 0.001	1	0000000000
118813002			-0.0162	0.0180	0.0132	-0.001 -0.001 -0.001		
118813003	GRAZ LUSTRUEHEL	SLR 7839	4194426.382	1162094.179	4647246.738	0.001 0.001 0.001	2	3510000000
118813004			-0.0162	0.0180	0.0132	-0.001 -0.001 -0.001		
118813005	GRAZ LUSTRUEHEL	SLR 7839	4194425.377	1162094.178	4647244.738	0.001 0.001 0.001	3	0000000000
118813006			-0.0162	0.0180	0.0132	-0.001 -0.001 -0.001		
118813007	Sofia	SLR 7848	4143999.892	1888896.898	4289824.827	1.026 1.085 0.984		
118813008			0.1094	0.0132	0.0133	1.098 1.172 0.979		
122895001	Borovick	SLR 7811	1738332.678	1140246.610	5821818.988	0.001 0.001 0.001	1	0000000000
122895002			-0.0173	0.0136	0.0048	-0.001 -0.001 -0.001		
122895003	Borovick	SLR 7812	1738332.687	1140246.615	5821818.998	0.001 0.001 0.001	2	0120000000
122895004			-0.0173	0.0136	0.0048	-0.001 -0.001 -0.001		

- Position and velocities in files :
- Post-seismic models in files:
- Seasonal parameters in CF or CM in files:

```

-----
Frequency 1 : 365.250 Combination at epoch 15: 1
-----
ALBH A 481290003 1 X 0.828 0.164 2.501 0.164
ALBH A 481290003 1 Y 0.715 0.225 3.045 0.224
ALBH A 481290003 1 Z -1.450 0.265 -3.333 0.265
ALGO A 481040002 1 X -0.249 0.189 -0.358 0.183
ALGO A 481040002 1 Y 1.917 0.268 3.077 0.242
ALGO A 481040002 1 Z -1.348 0.257 -2.237 0.239
BRMU A 425015004 1 X -0.495 0.171 -0.063 0.178
BRMU A 425015004 1 Y 0.688 0.386 0.558 0.382
BRMU A 425015004 1 Z 0.111 0.217 0.147 0.215
CASI A 660110001 1 X 1.131 0.098 0.175 0.098
CASI A 660110001 1 Y -0.189 0.137 -0.166 0.136
CASI A 660110001 1 Z 1.914 0.286 1.465 0.283
CASA A 484370002 1 X -0.446 0.426 0.631 0.437
CASA A 484370002 1 Y -1.338 0.666 2.981 0.685
CASA A 484370002 1 Z -0.051 0.564 -2.878 0.581
DRAO A 481850002 1 X 0.588 0.138 2.352 0.136
DRAO A 481850002 1 Y -0.462 0.284 3.464 0.281
DRAO A 481850002 1 Z -0.393 0.248 -3.948 0.257
FAIR A 484800001 1 X 0.192 0.179 0.198 0.169
FAIR A 484800001 1 Y -0.012 0.137 0.414 0.138
FAIR A 484800001 1 Z 0.316 0.376 -1.431 0.355
FORT A 416020001 1 X 2.177 0.388 -3.125 0.389
FORT A 416020001 1 Y -0.568 0.380 1.027 0.319
FORT A 416020001 1 Z 1.018 0.122 0.546 0.124
GODE A 40451M123 1 X -0.699 0.094 -0.748 0.093
GODE A 40451M123 1 Y 1.889 0.187 2.079 0.180
GODE A 40451M123 1 Z -1.059 0.158 -0.884 0.145
    
```

Frame transformation

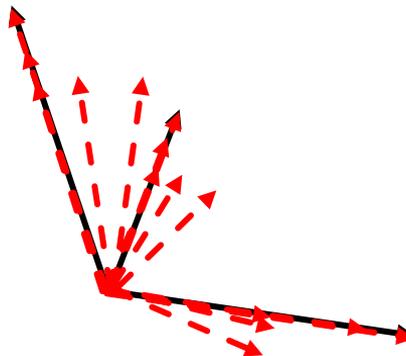
➤ You have all your results in ITRF2014 and want to transform them to ITRF2020?

3D similarity

Frame 1



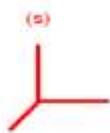
Frame 2



- Translation
- Rotation
- scale factor

7 parameters

$$X_1 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$$



\mathbf{M}



$$X_c = \begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix}$$

$$X_1 = [x_1, y_1, z_1]^T$$

$$X_2 = [x_2, y_2, z_2]^T$$

$$X_2 = T + \lambda R X_1$$



Small rotations

$$X_2 = T + (I_3 + d \cdot I_3 + R) X_1$$

14 parameter transformations

Relationship between dynamic TRF

14-parameter transformation

Velocities are frame-dependent!

$$\begin{pmatrix} x_2(t) \\ y_2(t) \\ z_2(t) \end{pmatrix} = \begin{pmatrix} t_x(t) \\ t_y(t) \\ t_z(t) \end{pmatrix} + \begin{pmatrix} x_1(t) \\ y_1(t) \\ z_1(t) \end{pmatrix} + d(t) \cdot \begin{pmatrix} x_1(t) \\ y_1(t) \\ z_1(t) \end{pmatrix} + \begin{pmatrix} 0 & -\epsilon_z(t) & \epsilon_y(t) \\ \epsilon_z(t) & 0 & -\epsilon_x(t) \\ -\epsilon_y(t) & \epsilon_x(t) & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ y_1(t) \\ z_1(t) \end{pmatrix}$$

$$\begin{pmatrix} \dot{x}_2(t) \\ \dot{y}_2(t) \\ \dot{z}_2(t) \end{pmatrix} = \begin{pmatrix} \dot{t}_x \\ \dot{t}_y \\ \dot{t}_z \end{pmatrix} + \begin{pmatrix} \dot{x}_1(t) \\ \dot{y}_1(t) \\ \dot{z}_1(t) \end{pmatrix} + \dot{d} \cdot \begin{pmatrix} x_1(t) \\ y_1(t) \\ z_1(t) \end{pmatrix} + \begin{pmatrix} 0 & -\dot{\epsilon}_z & \dot{\epsilon}_y \\ \dot{\epsilon}_z & 0 & -\dot{\epsilon}_x \\ -\dot{\epsilon}_y & \dot{\epsilon}_x & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ y_1(t) \\ z_1(t) \end{pmatrix}$$

With $t_x(t)$ $t_y(t)$ $t_z(t)$ $d(t)$ $\epsilon_x(t)$ $\epsilon_y(t)$ $\epsilon_z(t)$ \dot{t}_x \dot{t}_y \dot{t}_z \dot{d} $\dot{\epsilon}_x$ $\dot{\epsilon}_y$ $\dot{\epsilon}_z$ at $t=t_0$ } = the 14 parameters

* Rotation sign convention from the International Earth Rotation and Reference System Service (IERS).

14 parameter transformations

Example: transformation parameters from ITRF2014 to ETRF2014

Table 2: Transformation parameters from ITRF_{yy} to ETRF2014 **at epoch 2010.0** and their rates/year

ITRF Solution	T1 mm	T2 mm	T3 mm	D 10 ⁻⁹	R1 mas	R2 mas	R3 mas
ITRF2014	0.0	0.0	0.0	0.00	1.785	11.151	-16.170
rates	0.0	0.0	0.0	0.00	0.085	0.531	-0.770

Source: Altamimi, 2018, EUREF Technical Note 1: Relationship and Transformation between the International and the European Terrestrial Reference Systems

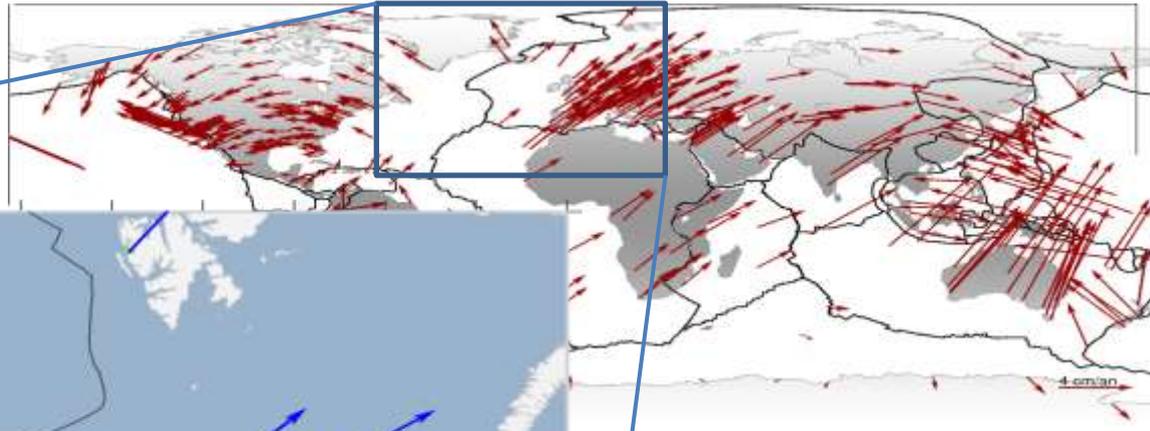
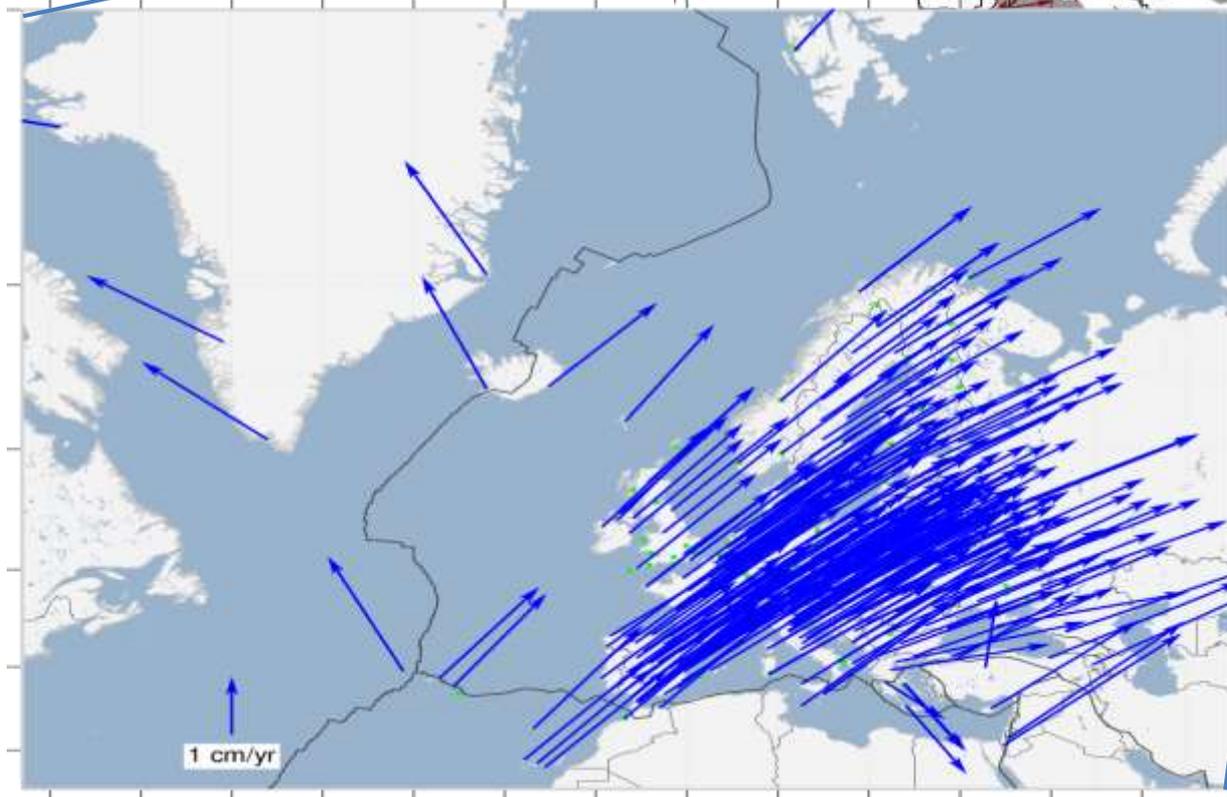
➔ Transformation parameter rates need to be used to compute the 7-parameter values at any epoch

Application: what are the rotations values in 1989.0 ?

$$\varepsilon_x(t) = R1(t) = R1(2010.0) + \dot{R1} \cdot (1989.0 - 2010.0) = 0$$

Transformation of velocities

Velocities in ITRF2014



Credit: Juliette Legrand

http://epncb.oma.be/_productservices/coordinates/posvel_map.php

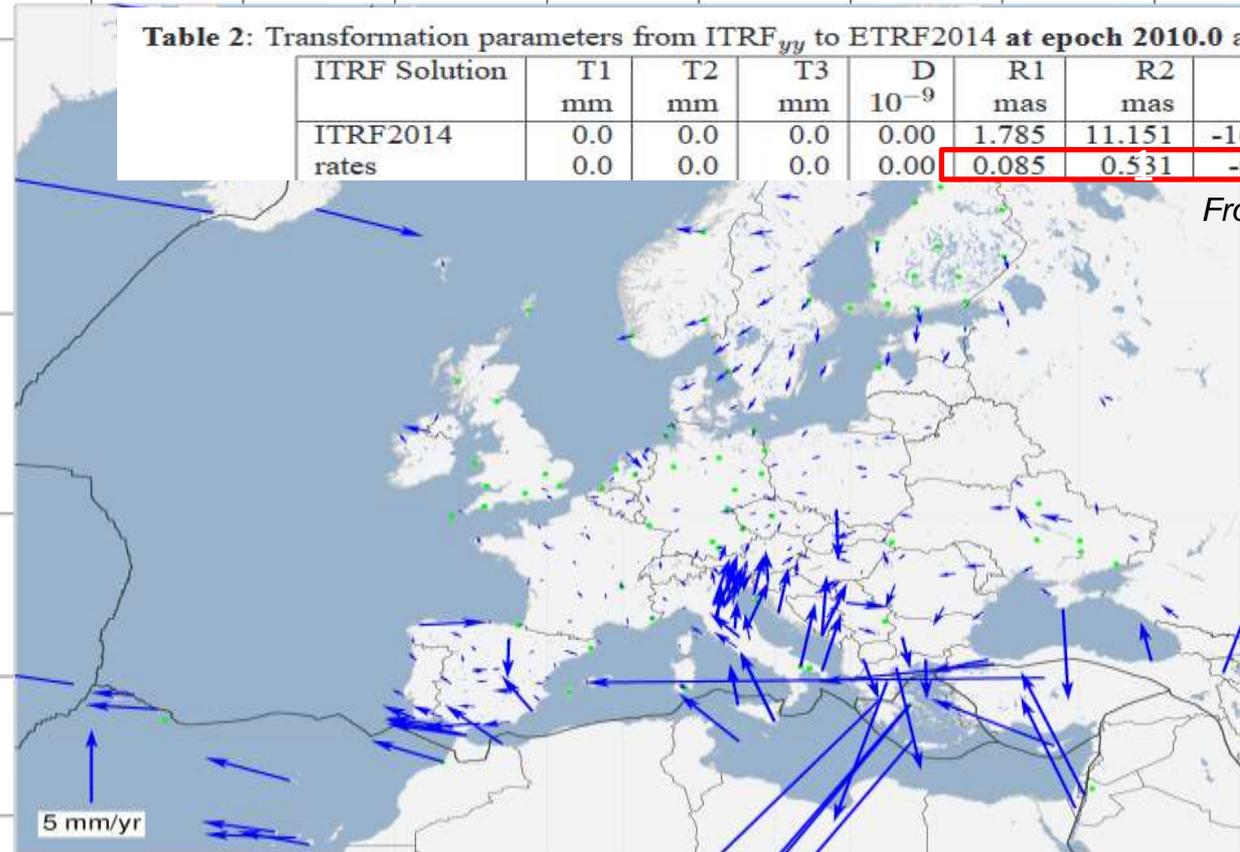
Transformation of velocities

Velocities in ETRF2014 after transforming velocities

Table 2: Transformation parameters from ITRF_{yy} to ETRF2014 at epoch 2010.0 and their rates/year

ITRF Solution	T1 mm	T2 mm	T3 mm	D 10 ⁻⁹	R1 mas	R2 mas	R3 mas
ITRF2014	0.0	0.0	0.0	0.00	1.785	11.151	-16.170
rates	0.0	0.0	0.0	0.00	0.085	0.531	-0.770

From Altamimi, (2018)



Credit: Juliette Legrand

http://epncb.oma.be/_productsservices/coordinates/posvel_map.php

14 parameter transformations

Transformation parameters from ITRF2020 to past ITRFs.

SOLUTION	Tx	Ty	Tz	D	Rx	Ry	Rz	EPOCH
UNITS----->	mm	mm	mm	ppb	.001"	.001"	.001"	
	
RATES	Tx	Ty	Tz	D	Rx	Ry	Rz	
UNITS----->	mm/y	mm/y	mm/y	ppb/y	.001"/y	.001"/y	.001"/y	
ITRF2014	-1.4	-0.9	1.4	-0.42	0.00	0.00	0.00	2015.0
rates	0.0	-0.1	0.2	0.00	0.00	0.00	0.00	
ITRF2008	0.2	1.0	3.3	-0.29	0.00	0.00	0.00	2015.0
rates	0.0	-0.1	0.1	0.03	0.00	0.00	0.00	
ITRF2005	2.7	0.1	-1.4	0.65	0.00	0.00	0.00	2015.0
rates	0.3	-0.1	0.1	0.03	0.00	0.00	0.00	
ITRF2000	-0.2	0.8	-34.2	2.25	0.00	0.00	0.00	2015.0
rates	0.1	0.0	-1.7	0.11	0.00	0.00	0.00	

Table available at <http://itrf.ign.fr>

- Successive Frame alignment in orientation
- Different scale definition strategy and modeling
- Translation rates get smaller, especially in TZ

14 parameter transformations

Online software to perform 14-parameter transformations between ITRF solutions and more:



EUREF Permanent GNSS Network



http://epncb.oma.be/products-services/coord_trans/

Home / Products & Services / Services / ETRF/ITRF Coordinate Transformation Tool (ECTT)

ETRF/ITRF Coordinate Transformation Tool (ECTT)

On-line coordinate transformation between coordinates (position and velocity) expressed in any ETRF realization of the [European Terrestrial Reference System \(ETRS89\)](#) and any ITRF realization of the [International Terrestrial Reference System \(ITRS\)](#). In case output coordinates are requested at a different epoch than the provided input coordinates, it is mandatory to also input station velocities.

For transformations to and from the Galileo Terrestrial Reference Frame (GTRF), use ITRF. GTRF is aligned to current versions of the ITRF.

Explanation and examples are available from the following [tutorial](#). However, note that with the introduction of the most recent transformation tool (August 2022), this tutorial has become slightly outdated.

Change epoch format:

INPUT

Frame: Epoch:

```
# Lines starting by # are treated as comments
# Fields (in decimal format) should be separated by at least one space
#
# --> Example without velocity -->
# Stationname (no space ch.) X[m] Y[m] Z[m]
Station_1 4812594.000 307845.000 4812474.010
#
# --> Example with velocity -->
# Stationname (no space ch.) X[m] Y[m] Z[m] VX[m/yr] VY[m/yr] VZ[m/yr]
Station_2 4812594.000 307845.000 4812474.010 0.01 0.2 0.02
```

TRANSFORM TO

Frame: Epoch:

Why choosing the most recent frame?

Effect of coordinate limited precision on Frame alignment?

- Note: $X_{ITRF}(t)$ becomes less precise with increasing t .
- Consequence: the « mean difference » between observed coordinates $X(t)$ and $X_{ITRF}(t)$ gets bigger with time !

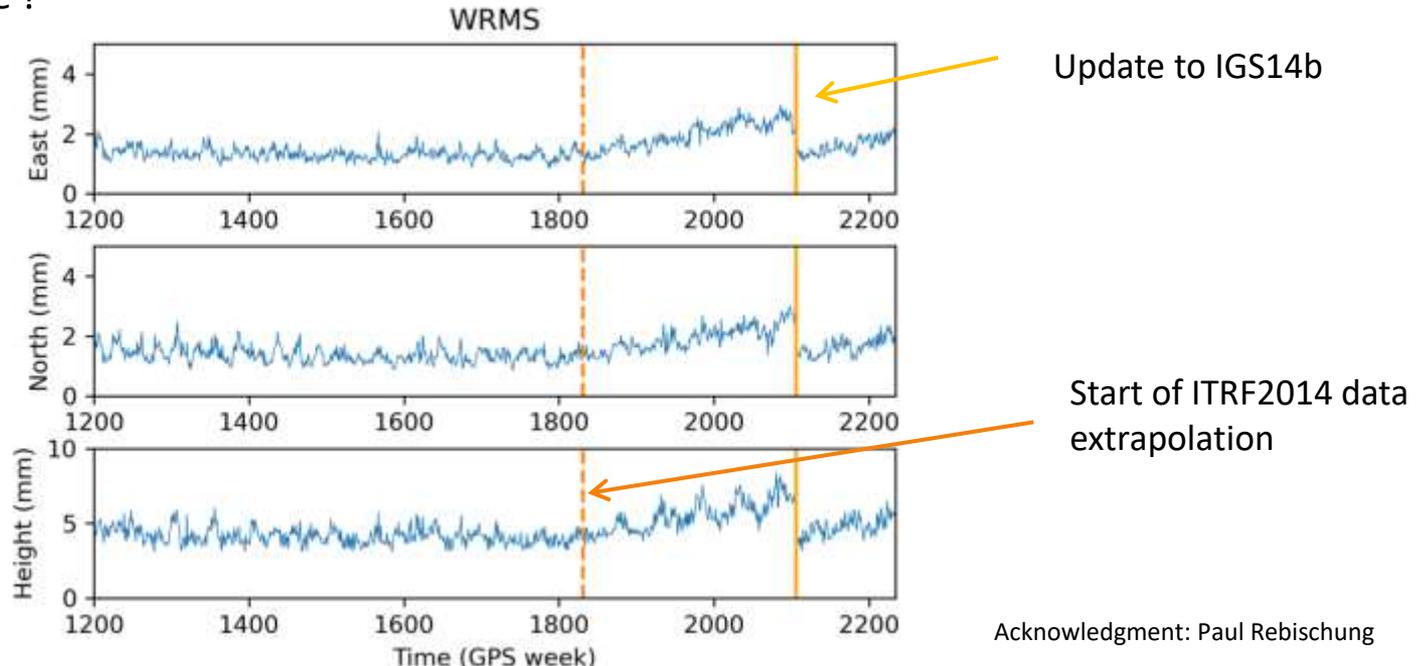
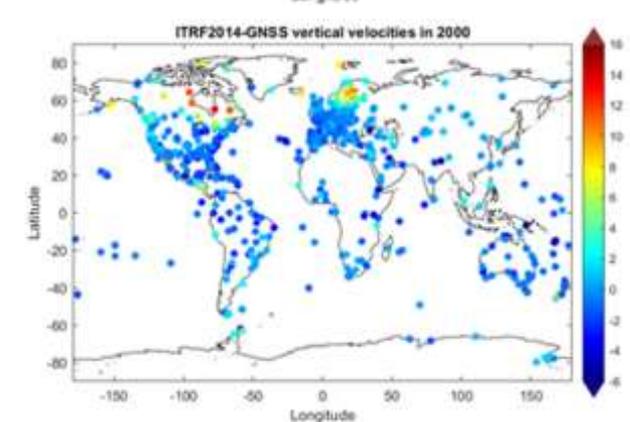
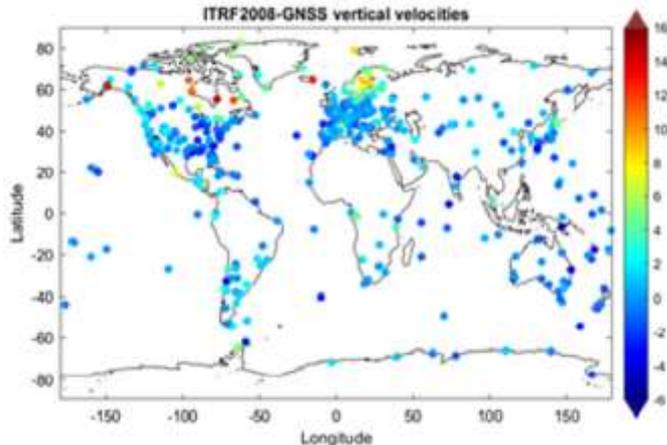
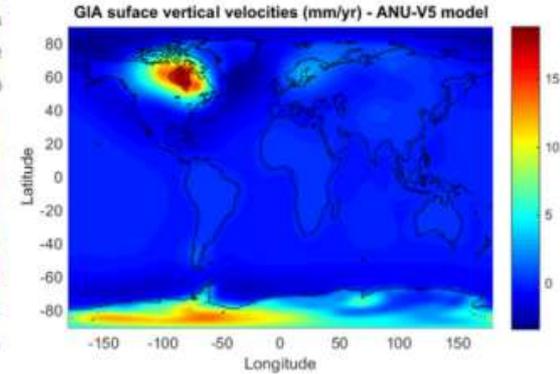
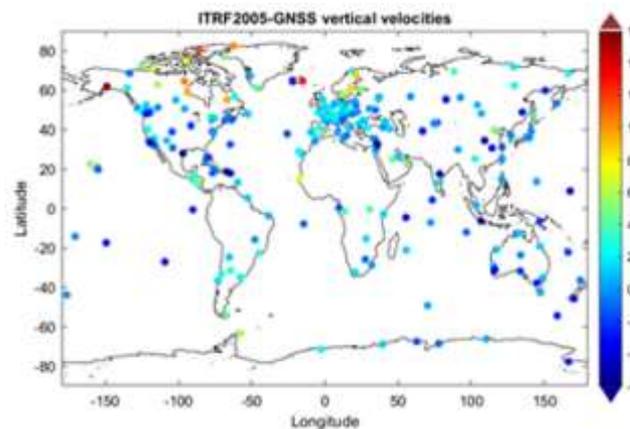
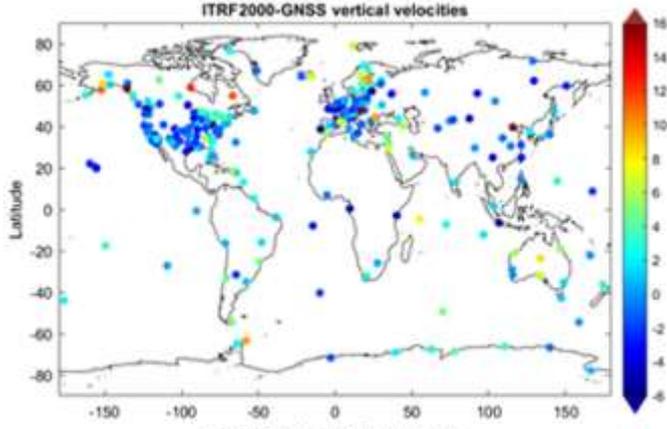


Fig. WRMS of 7-parameter estimation between GNSS weekly coordinate time series (IGS) and ITRF2014 (more precisely IGS14 and IGS14b frames)

Acknowledgment: Paul Rebischung

Why choosing the most recent frame?



Métivier, L., Altamimi, Z., & Rouby, H. (2020). Past and present ITRF solutions from geophysical perspectives. *Advances in space research*, 65(12), 2711-2722.

Frame accuracy

Frame accuracy:

- Frame datum accuracy
- individual station coordinate !

What accuracy for individual station coordinate?

- Agreement at ITRF2020 co-location sites

GNSS to	Tie vector discrepancies < 5 mm
VLBI	50% (=38 vectors)
SLR	36% (19 vectors)
DORIS	32% (39 vectors)

Altamimi Z, Rebischung P., Collilieux X., Métivier, L., Chanard, K., ITRF2020: An augmented reference frame refining the modeling of nonlinear station motions, J. Geodesy, submitted

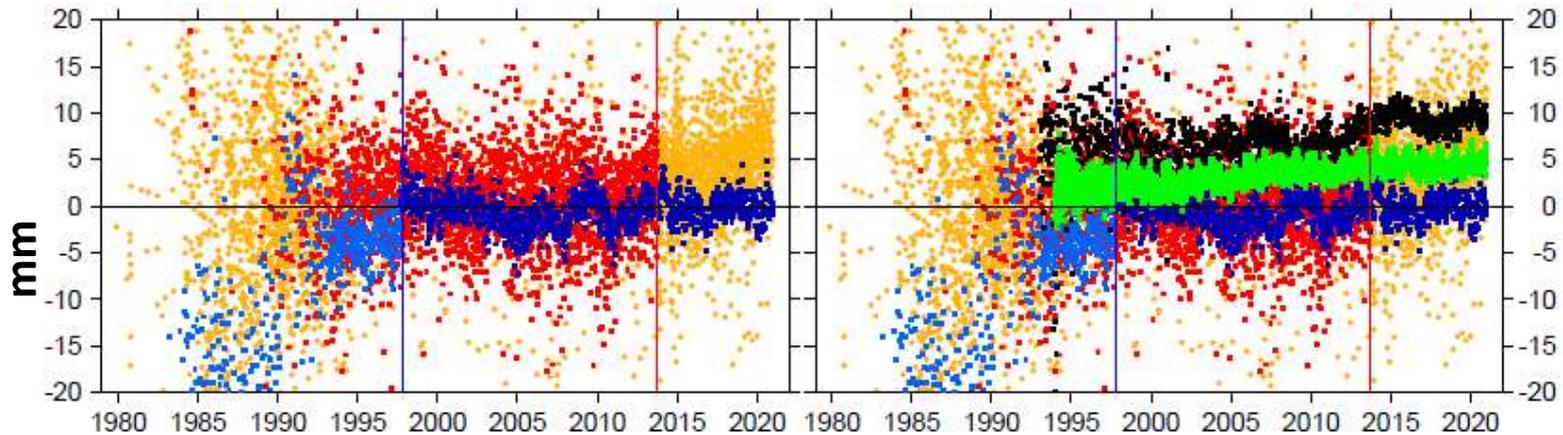


Frame accuracy

Frame datum accuracy

Internal = frame space geodesy results

Height positioning agreement between space geodetic techniques from ITRF2020 analysis



- **Orange and Red: VLBI scale**
- **Light blue and Dark blue: SLR scale**
- **Green: GNSS scale**
- **Black: DORIS scale**

Credit: Z. Altamimi

Altamimi, Z., P. Rebischung, X. Collilieux, L. Métivier and K. Chanard (2022)
ITRF2020: An overview of its features and results, REFAG 2022, oct 2022

Frame accuracy

Frame datum accuracy

External evaluation. Example of evaluation made on ITRF2008 origin time evolution

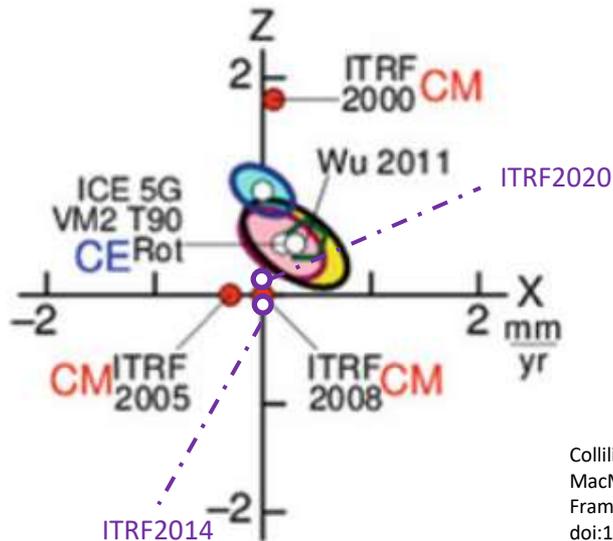


Fig. Evaluation of ITRF2008 origin drift along X and Z directions based on post-glacial rebound models and global inversion

Collilieux, X., Z. Altamimi, D.F. Argus, C. Boucher, A. Dermanis, B.J. Haines, T.A. Herring, C. Kreemer, F.G. Lemoine, C. Ma, D.S. MacMillan, J. Makinen, L. Métivier, J.C. Ries, F.N. Teferle and X. Wu (2014) External evaluation of the Terrestrial Reference Frame: report of the task force of the IAG sub-commission 1.2, International Association of Geodesy Symposia 139, pp. 197-202, doi:10.1007/978-3-642-37222-3_25

Thank you