ITRF2020P: Equations of post-seismic deformation models

After an Earthquake, the position of a station during the post-seismic trajectory, X_{PSD}, at an epoch t could be written as:

$$X_{PSD}(t) = X(t_0) + \dot{X}(t - t_0) + \delta X_{PSD}(t) \tag{1}$$

where $\dot{X}$ is the station linear velocity vector, and $\delta X_{PSD}(t)$ is the total sum of the post-seismic deformation (PSD) corrections at epoch t. For each component $L \in$ {E,N,U}, we note δL the total sum of PSD corrections expressed in the local frame at epoch t:

$$\delta L(t) = \sum_{i=1}^{n^l} A_i^l \log(1 + \frac{t - t_i^l}{\tau_i^l}) + \sum_{i=1}^{n^e} A_i^e (1 - \mathrm{e}^{-\frac{t - t_i^e}{\tau_i^e}}) \tag{2}$$

where:

n^l : Number of logarithmic terms of the parametric model
n^e : Number of exponential terms of the parametric model
A_i^l : Amplitude of the i^{th} logarithmic term
A_i^e : Amplitude of the i^{th} exponential term
τ_i^l: Relaxation time of the i^{th} logarithmic term
τ_i^e: Relaxation time of the i^{th} exponential term
t_i^l: Earthquake time(date) corresponding to i^{th} logarithmic term
t_i^e: Earthquake time(date) corresponding to the i^{th} exponential term

The variance of $\delta L(t)$ is given by:

$$\mathrm{var}(\delta L) = C.\mathrm{var}(\theta).C^T \tag{3}$$

where θ is the vector of parameters of the post-seismic deformation model:

$$\theta = [A_1^l, \tau_1^l, ..., A_{n^l}^l, \tau_{n^l}^l, A_1^e, \tau_1^e, ..., A_{n^e}^e, \tau_{n^e}^e]$$

The elements of the matrix C are computed by the following formulas:

$$\frac{\partial \delta L}{\partial A_i^l} = \log(1 + \frac{t - t_i^l}{\tau_i^l}) \tag{4}$$

$$\frac{\partial \delta L}{\partial \tau_i^l} = -\frac{A_i^l (t - t_i^l)}{(\tau_i^l)^2 (1 + \frac{t - t_i^l}{\tau_i^l})} \tag{5}$$

$$\frac{\partial \delta L}{\partial A_i^e} = 1 - \mathrm{e}^{-\frac{(t - t_i^e)}{\tau_i^e}} \tag{6}$$

$$\frac{\partial \delta L}{\partial \tau_i^e} = -\frac{A_i^e (t - t_i^e) \mathrm{e}^{-\frac{(t - t_i^e)}{\tau_i^e}}}{(\tau_i^e)^2} \tag{7}$$

Note that the PSD models are determined and provided to the users per component $L \in$ {E,N,U}, independently, and so there are NO cross-terms (or correlations) between components. However, cross-terms between amplitude and relaxation time for each LOG or/and EXP term should be taken into account in the variance calculation of equation (3). As an example, if for a given station there are 3 earthquakes that were taken into account in the estimation of the PSD models of its component E, and it has one EXP for the first EQ, one EXP for the 2nd EQ and LOG+EXP for the 3rd EQ, the one line matrix C for component E in equation (3) will have 8 terms.

Once the variances $\mathrm{var}(\delta E), \mathrm{var}(\delta N), \mathrm{var}(\delta U)$ are computed, the transformation into cartesian is obtained by:

$$\mathrm{var} \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} = R \, . \begin{bmatrix} \mathrm{var}(\delta E) & 0 & 0 \\ 0 & \mathrm{var}(\delta N) & 0 \\ 0 & 0 & \mathrm{var}(\delta U) \end{bmatrix} . \, R^T \tag{8}$$

where R is the transformation (Jacobian) matrix from topocentric to geocentric frame, and where:

$$\begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} = R \, . \begin{bmatrix} \delta E \\ \delta N \\ \delta U \end{bmatrix} \tag{9}$$